GOVERNMENT COLLEGE OF ENGINEERING, JALGAON

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Name of Examination : **Summer 2021** - (Preview)

Course Code & Course Name : **SH151U - Integral Calculus**

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Maximum Marks : **60**   Duration : **3 Hrs**

Edit   Print   View Answer Key   Close   **Answer Key Submission Type:** Marking scheme with model answers and solutions of numerical

Instructions:

1. *All questions are compulsory.*
2. *Illustrate your answer with suitable figures/sketches wherever necessary.*
3. *Assume suitable additional data; if required.*
4. *Use of logarithmic table, drawing instruments and non programmable calculators is allowed.*
5. *Figures to the right indicate full marks.*

1)   Solve all the questions.

a) Find the orthogonal trajectories of the family of parabola $x^2 = ay$.   [05]

b) Trace the curve $y^2(2a - x) = x^3$.   [05]

c) Show that $\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2}[erf(b) - erf(a)]$.   [04]

2)   Solve any three questions.

a) Solve $y\,e^y = \left(y^3 + 2x\,e^y\right)\frac{dy}{dx}$ [04]

b) Trace the curve $r = a\,\sin 3\theta$ [04]

c) Obtain half range cosine series for $f(x) = 1,\ 0 \le x \le 1,$ [04]
$$= x,\ 1 \le x \le 2$$

d) Evaluate $\int_0^1 \int_x^{\sqrt{x}} \left(x^2 + y^2\right) dx\,dy$ [04]

3)   Solve any three questions.

a) Solve $x\frac{dy}{dx} + y = y^2\,\log x$ [04]

b) Change the order of integration and hence evaluate   [04]
$$\int_0^1 \int_y^1 x^2\,e^{xy}\,dx\,dy$$

c) If $f(x) = \pi x,\ 0 \le x \le 1$ [04]
$$= \pi(x - 2),\ 1 \le x \le 2,\ \text{show that in this range } (0,\ 2)$$
$$f\left(x\right) = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx$$

d) Evaluate $\int_0^1 \left(x\,\log x\right)^4 dx$.   [04]

4)   Solve any three questions.

a) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\,dz\,dx\,dy$ [04]

b) Prove that for $-\pi < x < \pi$,   [04]
$$\frac{x\left(\pi^2 - x^2\right)}{12} = \frac{\sin x}{1^2} - \frac{\sin 2x}{2^2} + \frac{\sin 3x}{3^2} - \cdots\cdots$$

c) Find the total length of the curve $3a\,y^2 = x\,(x - a)^2$ [04]

d) Prove that $\int_0^{\infty} \frac{x^{m-1}}{(a+bx)^{m+n}}\,dx = \frac{1}{a^n\,b^m}\beta(m, n)$ [04]

5)   Solve any two questions.

a) A constant electromotive force E volts is applied to a circuit   [05]
containing a constant resistance R ohms in series and a
constant inductance L henries, if the initial current is zero,
show that the current builds up to half of its
theoretical maximum in $\frac{L\,\log 2}{R}\ seconds$.

b) Obtain the Fourier expansion of f(x)= cosax in the range $(0,\ 2\pi)$.   [05]

c) Evaluate $\iint xy\,dx\,dy\ over\ the\ region\ bounded\ by\ the\ x - axis,$ [05]
the line y =2 x and the parabola $y = \frac{x^2}{4a}$.

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