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 Name of Examination : **Summer 2021** - (Preview)

 Course Code & Course Name : **SH101U - Differential Calculus**

 Generated At : **19-04-2022 14:58:19**

 Maximum Marks : **60**

 Duration : **3 Hrs**
[Edit](#) [Print](#) [View Answer Key](#) [Close](#) **Answer Key Submission Type:** Marking scheme with model answers and solutions of numerical

Instructions:

1. All questions are compulsory.
2. Illustrate your answer with suitable figures/sketches wherever necessary.
3. Assume suitable additional data; if required.
4. Use of logarithmic table, drawing instruments and non programmable calculators is allowed.
5. Figures to the right indicate full marks.

**1) Solve all sub-questions**

 a) [5]

 If  $y = \sin^{-1} x$ , then show that

$$(1 - x^2) y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0.$$

 b) [4]

 If  $z^3 - xz - y = 0$ , then prove that  $\frac{\partial^2 z}{\partial x \partial y} = -\frac{(3z^2 + x)}{(3z^2 - x)^3}$ 

 c) [5]

Find the rank of matrix

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 37 \\ 3 & -1 & 1 & -1 & 6 \\ 2 & 3 & -1 & 4 & 23 \\ 5 & -1 & 0 & 1 & 18 \end{bmatrix}$$

**2) Solve any three sub-questions**

 a) [4]

 Prove that  $\cos^{-1} \left( \frac{3i}{4} \right) = \frac{\pi}{2} - i \log 2$ .

 b) [4]

 Find the total differential coefficient of  $x^2y$  with respect to  $x$ , if  $x$  and  $y$  are connected by the relation  $x^3 + y^3 + 3xy = 0$ .

 c) [4]

 Find the modulus and argument of  $i^{\log(1+i)}$ .

 d) [4]

 Express  $6 - x^2 - x^3 + 11x^4$  in powers of  $(x - 3)$ .

**3) Solve any three sub-questions**

 a) Divide 120 into three positive parts so that the sum of their products taken two at times shall be maximum. [5]

b) Examine the system for consistency and if consistent then solve it. [5]

$$\begin{aligned} 3x_1 + x_2 - 3x_3 &= -8; & 4x_1 + 2x_2 - x_3 &= -3 \\ x_1 + 2x_2 + 4x_3 &= 9; & 2x_1 + 4x_2 + 3x_3 &= 3 \end{aligned}$$

c) Show that the two circles [5]

$$\begin{aligned} x^2 + y^2 + z^2 - y + 2z &= 0, & x - y + z - 2 &= 0; \\ x^2 + y^2 + z^2 + x - 3y + z - 5 &= 0, & 2x - y + 4z - 1 &= 0; \end{aligned}$$

lie on the same sphere and find its equation.

d) Prove that  $\log\left(\frac{a+ib}{a-ib}\right) = 2i \tan^{-1} \frac{b}{a}$ . Hence evaluate  $\cos\left[i \log\left(\frac{a+ib}{a-ib}\right)\right]$ . [5]

4) a) Evaluate  $\lim_{x \rightarrow 0} (\cot x)^{\sin x}$  [4]

b) If  $u = \frac{x+y}{1-xy}$  and  $v = \tan^{-1} x + \tan^{-1} y$ , then show that  $u$  and  $v$  are functionally dependent. Also find relation between  $u$  and  $v$ . [5]

5) a) Show that the following matrix is orthogonal [6]

$$\begin{bmatrix} \cos \varphi \cos \theta & \sin \varphi & \cos \varphi \sin \theta \\ -\sin \varphi \cos \theta & \cos \varphi & -\sin \varphi \sin \theta \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$$

OR

a) Coordinates of a point P are (50, 50, 50). Origin is shifted to the point (5, -2, 3) with axes transferred parallel to themselves. Rotation is carried out about z-axis through  $45^\circ$ . Find the coordinates of the point P in new coordinate system. [6]

b) Convert the following equations to both (i) Spherical polar coordinates, (ii) Cylindrical polar coordinates. [4]

$$x = a - y^2 + 2xy, \quad y^2 + z^2 = b^2. \text{ Each conversion carries 1 mark.}$$